

Fun with Flow Handout

Students learn dimensional analysis while exploring the flow of history on the Wekiva River.

Dimensional Analysis

(Modified from the original lessons on the Seminole County Water Atlas, author Ron Browning.)

Units are a critical part of describing every measurement. Before you can work with units mathematically, you frequently must convert from one unit to another. Dimensional analysis does not do your math for you, but it helps to ensure that calculations are correct. After that, all you have to do is find the conversion factors and enter the numbers into a calculator.

Your Task

Conversion to metric units is important because the scientific community uses only metric units in their measurements and journals. Only a few countries, including the United States, cling to so-called "standard measurement units" as their official system. Conversion factors can be found on the website of the National Institute of Standards & Technology, <http://www.nist.gov/>. But first, look at the following information on using Dimensional Analysis for converting units. Additionally, science fair projects are only accepted using metric units.

Whatever You Measure, You Have to Use Units

When something is measured, the units should be specific and labeled. For instance, if I tell you I am 16 tall, you haven't learned much about my height: your proper response would be "16 what?" If I respond 16 inches, you know that I am a very short person; on the other hand, if I tell you I am 16 feet, you can sign me up for the basketball team.

The above examples are for units of length, but there are many other things that we measure, and all of them require units. For instance, time can be measured in seconds or minutes (or hours, days, etc); angles can be measured in degrees or in radians; and so on. Some units are made up of other units: for instance, you might measure the speed of a car in miles/hour. So "50 mph" means that every hour, the car travels 50 miles; very different from a car traveling 50 mi/s, for instance! No matter what is being measured, you must specify the units of measurement.

Sometimes Units Need to be Converted

Suppose that a 60-inch man stands on the head of a 6-foot man. How tall are they together? Obviously, you cannot find the answer to this question by adding 60 to 6. The reason the answer is not 66 is that the two figures are given in different units. Before you can add the two numbers, you have to convert one of them to the units of the other. Then, when you have two numbers in the same units, you can complete the calculation.

In order to perform this conversion, you need a conversion factor. That is, in this case, you have to know how many inches make up a foot. You probably already know the answer: 12 inches is 1 foot. So knowing that, you can perform the calculation in two steps as follows:

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- a. 60 inches is really $60/12 = 5$ feet
- b. 5 feet + 6 feet = 11 feet

The second half of the calculation is easy; once all of your units are the same, calculation is simple. The difficult portion is the conversion because sometimes you have to divide, and sometimes you have to multiply.

In the above example inches was converted to feet by dividing by 12. If I were going the other way, converting from feet to inches, I would have multiplied by 12. How do I figure out which to do?

One way is by using common sense. I know that 60 inches can't possibly be 720 feet; there obviously have to be fewer feet than inches. However, sometimes common sense fails. For instance, if you want to convert miles/hour into meters/second, what do you multiply or divide by what?

Fortunately, there is another relatively easy way to make conversions. This technique is called dimensional analysis.

Dimensional analysis tells you what to multiply or divide

Let's review the previous problem, but use dimensional analysis, just so we can introduce the method. As you recall, the interesting part of the problem was step a), where I had to convert 60 inches into feet. I knew that there are 12 inches in a foot, so common sense told me to divide by 12.

Here's the dimensional analysis way:

First of all, write the conversion factor as a fraction. 12 inches = 1 foot, so the conversion factor is

$$\frac{12 \text{ inches}}{1 \text{ foot}} \text{ or } \frac{1 \text{ foot}}{12 \text{ inches}}$$

Because 12 inches is the same thing as 1 foot, that fraction is just like 5/5; it's equal to 1. And we can multiply by it without changing anything.

So, let's try that on our 60 inches:

$$\frac{60 \text{ inches}}{1} * \frac{1 \text{ foot}}{12 \text{ inches}} = \frac{60}{12} = 5 \text{ feet}$$

The critical thing to note about the above equation is that the units behave like numbers do when you multiply fractions. That is, the inches on top and the inches on the bottom cancel out, leaving feet. Then all you have to worry about is the numbers.

Suppose I had written it the wrong way, like the following equation:

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$$\frac{60 \text{ inches}}{1} * \frac{12 \text{ inches}}{1 \text{ foot}} = 60 * 12 \frac{\text{inches} * \text{inches}}{\text{foot}}$$

I would have immediately realized that multiplying 60 * 12 is WRONG, because the units of the answer are inches*inches/feet instead of just feet. So the dimensional analysis shows me immediately that dividing is correct, and multiplying is not.

Such conversions are even easier in the metric system, of course, since all the conversion factors are multiples of ten. For instance, to convert 10 km to m, you would simply write:

$$10 \text{ km} * \frac{1000 \text{ m}}{1 \text{ km}} = 10,000 \text{ m}$$

At this point, you may well be asking yourself "so what?" Dimensional analysis doesn't take much time, and it isn't very difficult, but it is certainly more trouble than the common-sense method.

To answer that, let's look at a more difficult unit conversion problem. Suppose that I want to convert 55 mi/hr into m/s. I know that 1 mile is 5280 feet, that 1 meter is 3.3 feet, that 1 hour is 60 minutes, and that 1 minute is 60 seconds. Now, what do I multiply, or divide, by what?

This is a very tricky question, but using dimensional analysis, it becomes easy. First of all, I write all the above conversions as fractions, remembering that I might have to turn some of them upside-down.

$$\frac{1 \text{ mile}}{5280 \text{ ft}} \quad \frac{1 \text{ m}}{3.3 \text{ ft}} \quad \frac{1 \text{ hour}}{60 \text{ minutes}} \quad \frac{1 \text{ minute}}{60 \text{ seconds}}$$

Now we need units to cancel, so if we turn the feet/meters fraction around, we get:

$$\frac{55 \text{ miles}}{1 \text{ hour}} * \frac{5280 \text{ ft}}{1 \text{ mile}} * \frac{1 \text{ m}}{3.3 \text{ ft}} * \frac{1 \text{ hour}}{60 \text{ minutes}} * \frac{1 \text{ minute}}{60 \text{ seconds}}$$

So...

$$\frac{55 * 5280 \text{ m}}{(3.3 * 60 * 60) \text{ sec}}$$

$$= 24.4 \text{ m/sec}$$

If you think that was overly difficult, try to imagine what it would have been like without dimensional analysis! You would still have had to multiply the 55, 5280, 3.3, 60, and 60, but first you would have had to figure out which to multiply and divide. And if you had made a mistake, how would you have caught it?

For practice, use the above technique to make the conversion calculations for a dataset downloaded from the Water Atlas website. Switch from degrees Celsius to degrees Fahrenheit, feet to meters, or cubic feet per second to liters per second, for example.